Discussion
2/9/17

1.6.6
1.6.7 ✓
1.6.17 ✓
1.6.19 ✓
1.6.20
1.6.21
1.9.7 ✓
1.9.8 ✓
HW 1.9.10 (b)
1.9.13 ✓
1.9.14

1.6.7. A square matrix is called normal if it commutes with its transpose: $AA^T = A^TA$.

Find all normal 2×2 matrices.

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$AA^T = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{pmatrix}$$

$$A^TA = \begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2+c^2 & ab+cd \\ ab+cd & b^2+d^2 \end{pmatrix}$$

$a^2+b^2 = a^2+c^2 \Rightarrow b^2 = c^2$

$\Rightarrow b = \pm c$

$ab+cd = ac+bd \Rightarrow ab - bd = ac - cd$

$\Rightarrow b(a-d) = c(a-d)$

$\Rightarrow b = c$, if $a \neq d$.

Either $b = c$ or $a \neq d$ and $b = -c$.

1.6.17 (c) Find a, b, c for which

$$\begin{pmatrix} 3 & a+2b-2c & -4 \\ 6 & 7 & b-c \\ -a+b+c & 4 & b+3c \end{pmatrix}$$ is symmetric.

If symmetric,

$a+2b-2c = 6$

$-a+b+c = -4$

$b-c = 4$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 6 \\ -1 & 1 & 1 & -4 \\ 0 & 1 & -1 & 4 \end{array}\right]$$

$$\xrightarrow{R1+R2} \left[\begin{array}{cccc} 1 & 2 & -2 & 6 \\ 0 & 3 & -1 & 2 \\ 0 & 1 & -1 & 4 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{3}R2+R3} \left[\begin{array}{cccc} 1 & 2 & -2 & 6 \\ 0 & 3 & -1 & 2 \\ 0 & 0 & -2/3 & 3\,1/3 \end{array}\right]$$

$-\frac{2}{3}c = \frac{10}{3}$

$\Rightarrow \boxed{c = -5}$

$3b - (-5) = 2$

$\Rightarrow 3b = -3$

$\Rightarrow \boxed{b = -1}$

$a + 2(-1) - 2(-5) = 6$

$\Rightarrow a + 8 = 6$

$\Rightarrow \boxed{a = -2}$

check eq. #2 : $-(-2) + (-1) + (-5) = -4$ ✓

1.6.19 T or F : If A is symmetric, then A^2 is symmetric.

True : Assume $A = A^T$

then $A^2 = AA = A^T A^T$

$= (AA)^T$

$= (A^2)^T$

So A^2 is symmetric

1.9.7. Prove that similar matrices have the same determinant. i.e. $\det A = \det B$ whenever $B = S^{-1}AS$ for some matrix S.

" whenever = if "

Assume $B = S^{-1}AS$.

Then $\det B = \det(S^{-1}AS)$

$= \det S^{-1} \det A \det S$

$= \frac{1}{\det S} \det A \det S$

scalars

$= \frac{1}{\det S} \det S \; \det A$ since dets are scalars and so commute.

$= \det A$.

So $\det B = \det A$. ✓

1.9.8. Prove that if A is an $n \times n$ matrix and c is a scalar, then $\det(cA) = c^n \det A$

Assume A is $n \times n$ and c is scalar.

cA is obtained by multiplying each entry by c, or equivalenty multiplying each row by c.

multiplying a row by c causes the det to be mult. by c.

There are n rows in A, so the det is mult by c n times.

i.e. $\det(cA) = c^n \det A$.

1.9.13 Prove: For A nonsingular $\det A^{-1} = \frac{1}{\det A}$. using $\det(AB) = \det A \det B$

$\det(AB) = \det A \det B$

Then $\det(AA^{-1}) = \det A \det A^{-1}$

but $AA^{-1} = I$,

so $\det(AA^{-1}) = \det(I) = 1$.

thus $\det A \det A^{-1} = 1$

$\Rightarrow \det A^{-1} = \frac{1}{\det A}$, $\det A \neq 0$ since A is nonsingular.